ON REFINED CONJECTURES OF BIRCH AND SWINNERTON-DYER TYPE

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Let A be an abelian variety defined over a number field k. The conjecture of Birch and Swinnerton-Dyer ('BSD') predicts an explicit formula for the leading term at $z = 1$ of the Hasse-Weil L-series $L(A, z)$ of $A_{/k}$. Deligne and Gross have also predicted the order of vanishing at $z = 1$ of the Hasse-Weil-Artin L-series $L(A, \chi, z)$ associated to finite dimensional characters χ of the absolute Galois group of k.

In certain restricted settings conjectures of Mazur-Tate, resp. of Darmon, predict integral congruence relations between the values, resp. first derivatives, at $z = 1$ of the functions $L(A, \chi, z)$ as χ ranges over certain families of characters. Such congruence relations also involve the discriminants of canonical height pairings valued in finite groups. It has however proved much more difficult to formulate explicit refinements of BSD that take into account any connections that might exist between the leading terms of $L(A, \chi, z)$ for varying characters χ of arbitrary order of vanishing.

Let F be a finite Galois extension of k. In this talk we present a completely general 'refined BSD conjecture' for $(A, F/k)$ and show that it provides an appropriate framework for the investigation of such connections as χ ranges over the irreducible characters of $\mathrm{Gal}(F/k)$.

Our refined BSD conjecture is consistent with the relevant case of the equivariant Tamagawa number conjecture and in particular renders the latter conjecture amenable to theoretical or numerical verifications in situations in which it encodes a genuine mixture of archimedean considerations (related to the computation of Néron-Tate heights) and of p-adic congruence relations (related to the computation of Mazur-Tate heights).

This is joint work with D. Burns.